QUARK MASS EFFECTS IN DEEPLY INELASTIC SCATTERING

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Abstract

We argue that the difference between the structure functions corresponding to deep inelastic scattering with and without heavy quarks in the current fragmentation region tends to a constant value at large Q^2 and fixed (low) x_{Bj}.

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1 Introduction

Quite often mass effects in high–energy collisions are considered as some not very spectacular corrections which finally die off. Nonetheless, it appears that in e^+e^- annihilation even such overall characteristics as hadron multiplicities are quite sensitive to the value of masses of the primary $q\bar{q}$ pairs [1].

Recent considerations have shown that calculations based on QCD agree well with the data at high enough energy [2] and that they yield an asymptotically constant difference between multiplicities of hadrons induced by the primary quarks of different masses.

In this paper we consider the possibility of a similar effect in a deeply inelastic process.

2 Calculation of quark mass dependence

Let us consider, for definiteness, deep inelastic scattering of the electron (muon) off the proton. The hadronic tensor (an imaginary part of the virtual photon–proton amplitude) is defined via the electromagnetic current J_μ:

$$W_{\mu\nu}(p,q) = \frac{1}{2}(2\pi)^2 \int d^4z \exp(iqz)\langle p|[J_\mu(z), J_\nu(0)]|p\rangle, \tag{1}$$

where p is the momentum of the proton, $p^2 = M^2$, and q is the momentum of virtual photon, $q^2 = -Q^2 < 0$.

A symmetric part of $W_{\mu\nu}$ has two Lorentz structures:

$$W_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\nu q_\nu}{q^2}\right) F_1(Q^2, x) + \frac{1}{pq}\left(p_\mu - q_\mu \frac{pq}{q^2}\right)\left(p_\nu - q_\nu \frac{pq}{q^2}\right) F_2(Q^2, x), \tag{2}$$

where the structure functions F_1 and F_2 depend on Q^2 and on the variable

$$x = \frac{Q^2}{pq + \sqrt{(pq)^2 + Q^2M^2}}. \tag{3}$$

In what follows we will analyse the structure function F_2 of deep inelastic scattering with open charm (beauty) production at small x. In this section we consider the case of one single quark loop with mass m_q and electric charge e_q. A general case will be discussed in Section 2.

At small x a leading contribution to F_2 comes from one photon–gluon fusion subprocess [4]:

$$W_{\mu\nu} = \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^4} C^{\alpha\beta}_{\mu\nu}(q, k; m_q) d_{\alpha\alpha'}(k) d_{\beta\beta'}(k) \Gamma^{\alpha'\beta'}(k, p), \tag{4}$$

where k is the momentum of the virtual gluon, $k^2 < 0$. The tensor $C^{\alpha\beta}_{\mu\nu}$ denotes an imaginary two gluon irreducible part of the photon–gluon amplitude, while $\Gamma^{\alpha\beta}$ describes a distribution of the gluon inside the proton. A quantity $d_{\alpha\beta}$ is a tensor part of the gluonic propagator.

Let us choose an infinite momentum frame ($P \to \infty$)

$$p_\mu = \Bigl(P + \frac{M^2}{4P}, 0, 0, P - \frac{M^2}{4P}\Bigr), \tag{5}$$

that is $p_\mu \simeq (1,0,0,1)$. Then the gluon distribution $\Gamma^{\alpha\beta}$ has to be calculated in the axial gauge $nA = 0$ with a gauge vector $n_\mu = (1,0,0,-1)$ [4]. One can take, for instance,

$$n_\mu = q_\mu + x p_\mu \tag{6}$$

with x defined by Eq. (3).

From Eq. (2) we get

$$\frac{1}{x}F_2 = \Bigl[-g_{\mu\nu} + p_\mu p_\nu \frac{3Q^2}{(pq)^2 + Q^2M^2}\Bigr] W^{\mu\nu} \equiv F_2^{(a)} + F_2^{(b)}. \tag{7}$$

Two terms in the RHS of Eq. (7), $F_2^{(a)}$ and $F_2^{(b)}$, correspond to two tensor projectors, $g_{\mu\nu}$ and $p_\mu p_\nu$.

Note that the structure function $F_L = F_2 - 2xF_1$ is completely defined by the term $p_\mu p_\nu$ and, thus, proportional to $F_2^{(b)}$.

By definition, the gluon distribution $\Gamma^{\alpha\beta}$ can be rewritten in the form

$$\Gamma^{\alpha\beta} = \frac{1}{4\pi}\sum_n \delta(p + k - p_n)\langle p|I^g_\alpha(0)|n >< n|I^g_\beta(0)|p\rangle, \tag{8}$$

where I^g_α is the conserved current. Both $|p\rangle$ and $|n\rangle$ are on shell states that result in

$$k^\alpha \Gamma_{\alpha\beta} = 0. \tag{9}$$

From an explicit form for $C^{\alpha\beta}_{\mu\nu}$ (see Appendix I) one can verify that it obeys the same condition:

$$k^\alpha C^{\mu\nu}_{\alpha\beta} = 0. \tag{10}$$

Equations (9) and (10) allow us to simplify expression (4) and get ($r = a, b$):

$$\frac{1}{x}F_2^{(r)} = \int \frac{d^4k}{(2\pi)^4}\frac{1}{k^4} C^{(r)}_{\alpha\beta}(q,k;m_q)\Gamma^{\alpha\beta}(k,p), \tag{11}$$

with the notations

$$\begin{aligned} C^{(a)}_{\alpha\beta} &= -g_{\mu\nu}C^{\mu\nu}_{\alpha\beta}, \\ C^{(b)}_{\alpha\beta} &= \frac{3Q^2}{(pq)^2 + Q^2M^2} p_\mu p_\nu C^{\mu\nu}_{\alpha\beta}. \end{aligned} \tag{12}$$

The tensor $\Gamma^{\alpha\beta}$ can be expanded in Lorentz structures

$$\begin{aligned}\Gamma^{\alpha\beta} &= \left(g_{\alpha\beta} - \frac{k_\alpha k_\beta}{k^2}\right)\Gamma_1 + \left(p_\alpha - k_\alpha \frac{pk}{k^2}\right)\left(p_\beta - k_\beta \frac{pk}{k^2}\right)\frac{1}{k^2}\Gamma_2 \\ &+ \left(k_\alpha - n_\alpha \frac{k^2}{kn}\right)\left(k_\beta - n_\beta \frac{k^2}{kn}\right)\frac{1}{k^2}\Gamma_3 + \left(p_\alpha - n_\alpha \frac{pk}{kn}\right)\left(p_\beta - n_\beta \frac{pk}{kn}\right)\frac{1}{k^2}\Gamma_4 \end{aligned} \tag{13}$$

with $\Gamma_i = \Gamma_i(k^2, M^2, pk)$.

Let us consider a contribution of the invariant function Γ_1 into the structure function F_2 (11). With the accounting for (9) and (10) we obtain

$$\frac{1}{x}F_2^{(r)} = e_q^2 \int\limits_x^1 \frac{dz}{z} \int\limits_{Q_0^2}^{Q^2(z/x)} \frac{dl^2}{l^2} \frac{1 - l^2x^2/Q^2z^2}{1 + M^2x^2/Q^2} C^{(r)}\left(\frac{Q^2}{l^2}, \frac{m_q^2}{l^2}, \frac{x}{z}\right) \frac{\partial}{\partial \ln l^2} g(l^2, z), \tag{14}$$

where

$$l^2 = -k^2 > 0, \tag{15}$$

$$z = \frac{kn}{pn} \tag{16}$$

and

$$Q_0^2 = \frac{M^2z^2}{1 - z}. \tag{17}$$

Here we used the notation:

$$C^{(r)} = -g^{\alpha\beta}C_{\alpha\beta}^{(r)}. \tag{18}$$

In Eq. (14) the gluon distribution, $g(l^2, z)$, is introduced:

$$g(l^2, z) = \frac{1}{2(2\pi)^4} \int\limits_{Q_0^2}^{l^2} \frac{dl'^2}{l'^4} \int d^2k_\perp \Gamma_1(l'^2, k_\perp, z). \tag{19}$$

If we use the new variable

$$\xi = \frac{-k^2}{pk + \sqrt{(pk)^2 - k^2M^2}} \tag{20}$$

instead of $k_\perp^2$, we will arrive at the expression

$$g(l^2, z) = \frac{z}{32\pi^3} \int\limits_{Q_0^2}^{l^2} \frac{dl'^2}{l'^4} \int\limits_z^1 d\xi \left(M^2 + \frac{l'^2}{\xi^2}\right) \Gamma_1(l'^2, \xi). \tag{21}$$

A thorough analysis shows, however, that the main contribution to F_2 at small x comes from Γ_2 and Γ_4 in (13) (see also Appendix II). In Appendix I the following formula for F_2 is obtained:

$$\frac{1}{x}F_2 = e_q^2 \sum_{r=a,b} \int_z^1 \frac{dz}{z} \int_{Q_0^2}^{Q^2(z-x)/x} \frac{dl^2}{l^2} \quad \left[\tilde{C}^{(r)}\left(\frac{Q^2}{l^2}, \frac{m_q^2}{l^2}, \frac{x}{z}\right) \frac{\partial}{\partial \ln l^2} G(l^2, z) \right.$$
$$\left. + \hat{C}^{(r)}\left(\frac{Q^2}{l^2}, \frac{m_q^2}{l^2}, \frac{x}{z}\right) \frac{\partial}{\partial \ln l^2} \hat{G}(l^2, z)\right]. \qquad (22)$$

As we are interested in a calculation of the difference of the structure functions corresponding to the massive and massless cases, we preserve those terms in $C^{(r)}$ which give a leading contribution to ΔF_2. In Appendix I we have calculated the functions $C^{(a)}$ in lowest order in the strong coupling α_s:

$$\begin{aligned} \tilde{C}^{(a)}(u,v,y) &= \frac{\alpha_s}{4\pi}\{[(1-y)^2+y^2]L(u,v,y) - [(1-y)^2+y^2-2v]M(v,y) - 1\}, \\ \hat{C}^{(a)}(u,v,y) &= \frac{\alpha_s}{\pi} y(1-y)M(v,y), \end{aligned} \qquad (23)$$

where

$$\begin{aligned} L(u,v,y) &= \ln\frac{u(1-y)}{y[v+y(1-y)]}, \\ M(v,y) &= \frac{y(1-y)}{v+y(1-y)}. \end{aligned} \qquad (24)$$

As for the gluon distributions, they are given by the formulae:

$$G = \frac{1}{32\pi^3 z} \int_{Q_0^2}^{l^2} \frac{dl'^2}{l'^4} \int_z^1 \frac{d\xi}{\xi}(\xi - z)\left(M^2 + \frac{l'^2}{\xi^2}\right)[\Gamma_2(l'^2,\xi) + \Gamma_4(l'^2,\xi)], \qquad (25)$$

$$\hat{G} = \frac{1}{32\pi^3 z} \int_{Q_0^2}^{l^2} \frac{dl'^2}{l'^4} \int_z^1 d\xi \left(M^2 + \frac{l'^2}{\xi^2}\right)\left[\frac{(2\xi - z)^2}{4\xi^2}\Gamma_2(l'^2,\xi) + \Gamma_4(l'^2,\xi)\right]. \qquad (26)$$

The analogous expressions for the functions $C^{(b)}$ are the following:

$$\begin{aligned} \tilde{C}^{(b)}(u,v,y) &= \frac{3\alpha_s}{\pi}\frac{1}{u}y^2\{[2(1-2y)(1-y) - 2v]L(u,v,y) \\ &+ (1-y)[(1-y)^2+y^2-2v]M(v,y)\} + \frac{3\alpha}{2\pi}y(1-y), \\ \hat{C}^{(b)}(u,v,y) &= -\frac{12\alpha_s}{\pi}\frac{1}{u}y^2(1-y)^2M(v,y). \end{aligned} \qquad (27)$$

It may be shown that the leading contribution to ΔF_2 comes from the region $l^2 \sim m_q^2$, $k^2 = -l^2$ being the gluon virtuality (see Appendix I). Then one can easily see from (24) and

(28) that the first two terms in $\tilde{C}^{(b)}$ are suppressed by the factor k^2/Q^2 with respect to $\tilde{C}^{(a)}$, while the third terms in $\tilde{C}^{(b)}$ do not contribute to the difference $C^{(b)}|_{m=0} - C^{(b)}|_{m\neq 0}$.

In the leading logarithmic approximation (LLA), only the function L remaines in Eqs. (23), which results in

$$\frac{1}{x}\frac{\partial}{\partial \ln Q^2}F_2(Q^2,x) = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}P_{qg}\Big(\frac{x}{z}\Big)G(Q^2,z), \tag{28}$$

where $P_{qg}(z)$ is the Altarelli–Parisi splitting function and $G(Q^2,z)$ is the gluon distribution in LLA defined by Eq. (25).

The gluon distribution (25) in LLA can be rewritten in the form (compare it with the corresponding formulae in Appendix II)

$$G(Q^2,z) = \frac{1-z}{4z^2}\int_{Q_0^2}^{Q^2}\frac{dl^2}{l^4}\int\frac{d^2k_\perp}{(2\pi)^4}[\Gamma_2(l^2,k_\perp,z)+\Gamma_4(l^2,k_\perp,z)]. \tag{29}$$

It is clear from (22) that $\Delta F_2 = F_2|_{m=0} - F_2|_{m\neq 0}$ is defined by the quantities ($r=a,b$)

$$\Delta C^{(r)}(u,v,y) = C^{(r)}(u,0,y) - C^{(r)}(u,v,y). \tag{30}$$

By using Eq. (23) we obtain the important result

$$\begin{aligned}\Delta\tilde{C}^{(a)} &= \Delta\tilde{C}^{(a)}(v,y)\\ \Delta\hat{C}^{(a)} &= \Delta\hat{C}^{(a)}(v,y),\end{aligned} \tag{31}$$

while from (27) we get

$$\begin{aligned}\Delta\tilde{C}^{(b)} &= \frac{1}{u}\Delta\tilde{C}^{(b)}(v,y),\\ \Delta\hat{C}^{(b)} &= \frac{1}{u}\Delta\hat{C}^{(b)}(v,y).\end{aligned} \tag{32}$$

In this, we have

$$\Delta\tilde{C}^{(a)},\Delta\hat{C}^{(a)}|_{-k^2\to\infty} \sim \frac{m_q^2}{k^2}. \tag{33}$$

So, we get [3]

$$\begin{aligned}\frac{1}{x}\Delta F_2(Q^2,m_q^2,x)|_{Q^2\to\infty} = e_q^2\int_x^1\frac{dz}{z}\int_{Q_0^2}^{\infty}\frac{dl^2}{l^2}\quad &\Big[\Delta\tilde{C}\left(\frac{m_q^2}{l^2},\frac{x}{z}\right)\frac{\partial}{\partial\ln l^2}G(l^2,z)\\ &+\Delta\hat{C}\left(\frac{m_q^2}{l^2},\frac{x}{z}\right)\frac{\partial}{\partial\ln l^2}\hat{G}(l^2,z)\Big].\end{aligned} \tag{34}$$

Here

$$\begin{aligned}\Delta\tilde{C}(v,y) &= \frac{\alpha_s}{4\pi}\Big\{[(1-y)^2+y^2]\ln\Big[1+\frac{v}{y(1-y)}\Big] - \frac{v}{v+y(1-y)}\Big\},\\ \Delta\hat{C}(v,y) &= \frac{\alpha_s}{\pi}y(1-y)\frac{v}{v+y(1-y)}\end{aligned} \tag{35}$$

with $G(l^2, z)$ and $\hat{G}(l^2, z)$ being defined by Eqs. (25) and (26).

The integral in l^2 (34) converges because of condition (33). Contributions from $\Delta\tilde{C}^{(b)}$ and $\Delta\hat{C}^{(b)}$ are suppressed by the factors $(m^2/Q^2)\ln Q^2$ and can thus be omitted.

Let us consider the gluon distribution $\hat{G}$ (26). At small z the leading contribution to $\hat{G}(l^2, z)$ comes from the region $z \ll \xi$, and we have

$$\hat{G}(l^2, z) \simeq G(l^2, z). \tag{36}$$

Taking expression (36) into account, the structure function F_2 (22) has the following form at low x (with the term of the order of k^2/Q^2 and m^2/Q^2 subtracted)

$$\frac{1}{x}F_2 = e_q^2 \int_x^1 \frac{dz}{z} \int_{Q_0^2}^{Q^2(z-x)/x} \frac{dl^2}{l^2} C\left(\frac{Q^2}{l^2}, \frac{m_q^2}{l^2}, \frac{x}{z}\right) \frac{\partial}{\partial \ln l^2} G(l^2, z), \tag{37}$$

where

$$C(u, v, y) = \frac{\alpha_s}{4\pi}\{[(1-y)^2 + y^2]L(u, v, y) - [(1-3y)^2 - 3y^2 - 2v]M(v, y) - 1\}. \tag{38}$$

As for the difference of the structure function, we obtain the following prediction

$$\frac{1}{x}\Delta F_2(Q^2, m_q^2, x) = \frac{1}{x}\Delta F_2(m_q^2, x) = e_q^2 \int_x^1 \frac{dz}{z} \int_{Q_0^2}^{\infty} \frac{dl^2}{l^2} \Delta C\left(\frac{m_q^2}{l^2}, \frac{x}{z}\right) \frac{\partial}{\partial \ln l^2} G(l^2, z), \tag{39}$$

where

$$\Delta C(v, y) = \frac{\alpha_s}{4\pi}[(1-y)^2 + y^2]\left\{\ln\left[1 + \frac{v}{y(1-y)}\right] - (1-2y)^2 \frac{v}{v + y(1-y)}\right\}. \tag{40}$$

3 Relation between measurable structure functions

Up to now, we considered those contributions to F_2 that came from the quark with electric charge e_q and mass m_q, $\tilde{F}_2|_{m\neq 0}$. Then we have taken the analogous contributions from the massless quark with the same e_q, $\tilde{F}_2|_{m=0}$, and have calculated the quantity $\tilde{F}_2|_{m=0} - \tilde{F}_2|_{m\neq 0}$.

The total structure function F_2 has the form

$$F_2(Q^2, x) = \sum_q e_q^2 \tilde{F}_2^q(Q^2, x), \tag{41}$$

where the functions $\tilde{F}_2^q$ are introduced ($q = u, d, s, c, b$).

The structure functions describing open charm and bottom production in DIS, F_2^c and F_2^b respectively, are related to $\tilde{F}_2^c$ and $\tilde{F}_2^b$ by the formulae

$$\begin{aligned} F_2^c &= \frac{4}{9}\tilde{F}_2^c, \\ F_2^b &= \frac{1}{9}\tilde{F}_2^b. \end{aligned} \tag{42}$$

At low x one can put ($m_u = m_d = m_s = 0$ is assumed)

$$\tilde{F}_2^u = \tilde{F}_2^d = \tilde{F}_2^s = \tilde{F}_2 \tag{43}$$

and define the difference between heavy and light flavour contributions to F_2:

$$\begin{aligned} \Delta\tilde{F}_2^c = \tilde{F}_2 - \tilde{F}_2^c, \\ \Delta\tilde{F}_2^b = \tilde{F}_2 - \tilde{F}_2^b. \end{aligned} \tag{44}$$

Notice that there are the functions $\tilde{F}_2$ and $\tilde{F}_2^q$ that have been calculated in the previous section (see Eqs (37) and (39)).

Let us now represent the function $\tilde{F}_2$ (37) in the following form

$$\frac{1}{x}\tilde{F}_2 = \int\limits_x^1 \frac{dy}{y} \int\limits_0^Y d\eta\, C(\eta, y) \frac{\partial}{\partial \ln l^2} G\left(Y - \eta, \frac{x}{y}\right), \tag{45}$$

where we denote

$$Y = \ln \frac{Q^2}{yQ_0^2} \tag{46}$$

and introduce the variable $\eta = \ln(k^2/Q_0^2)$.

Analogously, we get from (39)

$$\frac{1}{x}\Delta\tilde{F}_2^q = \int\limits_x^1 \frac{dy}{y} \int\limits_{-\infty}^{Y_m} d\eta\, \Delta C(\eta, y) \frac{\partial}{\partial \ln l^2} G\left(Y_m - \eta, \frac{x}{y}\right), \tag{47}$$

with

$$Y_m = \ln \frac{m_q^2}{yQ_0^2}. \tag{48}$$

Here $\eta = \ln(m_q^2/k^2 y(1-y)) \simeq \ln(m_q^2/k^2 y)$ (remember that we consider small x).

The expression for ΔC is given by Eq. (40) and, in terms of the variables η and y, looks like

$$\Delta C = \frac{\alpha_s}{4\pi}[(1-y)^2 + y^2]\left[\ln(1 + e^\eta) - (1-2y)^2 \frac{e^\eta}{1 + e^\eta}\right]. \tag{49}$$

As for the expression for C, it has to be defined via relation (11) and exact formulae (I.14) and (I.21) taken at $m=0$. The result of our calculations is of the form

$$C(\eta,y)=\frac{\alpha_s}{2\pi}\left[\frac{1}{2U}\ln\frac{1+U}{1-U}\left(1-\frac{3}{U^2}V+V\right)-\left(1-\frac{3}{U^2}V\right)\right], \tag{50}$$

where

$$\begin{aligned} U &= \sqrt{1-4y(1-y)e^{-\eta}}, \\ V &= (1-y)\left[y+(1-y)e^{-\eta}\right]\left(1-e^{-\eta}\right). \end{aligned} \tag{51}$$

It is clear from (49) that

$$\Delta C(\eta,y)>0 \tag{52}$$

for $-\infty<\eta<\infty$, $0\le y\le 1$, while $\Delta C(\eta,y)$ is negligible at $\eta<0$ (see Figs. 1a-1d).

Moreover, the quantitative analysis shows that at most at $y\le 0.2$, which is relevant for small x as under consideration, one has

$$C(\eta,y)>\Delta C(\eta,y), \qquad \eta>0 \tag{53}$$

(see Figs. 2a-2d). Neglecting the small contribution to $\tilde{F}_2$ from the region $\eta<0$ and taking into account that $\partial G(Q^2,x)/\partial\ln Q^2>0$ at small x (cf. [5]), we thus conclude

$$\Delta\tilde{F}_2^q(m_q^2,x)<\tilde{F}_2(Q^2,x)|_{Q^2=m_q^2}. \tag{54}$$

In terms of the observables F_2, F_2^c and F_2^b, the inequalities (52) and (54) can be cast in the forms

$$\begin{aligned} \left(F_2-2.5F_2^c-F_2^b\right)(Q^2,x)|_{\text{large }Q^2} &> 0, \\ \left(F_2-F_2^c-7F_2^b\right)(Q^2,x)|_{\text{large }Q^2} &> 0 \end{aligned} \tag{55}$$

and

$$\begin{aligned} \left(F_2-2.5F_2^c-F_2^b\right)(Q^2,x) &< \left(F_2-F_2^c-F_2^b\right)(Q^2,x)|_{Q^2=m_c^2}, \\ \left(F_2-F_2^c-7F_2^b\right)(Q^2,x) &< \left(F_2-F_2^c-F_2^b\right)(Q^2,x)|_{Q^2=m_b^2}. \end{aligned} \tag{56}$$

Data on the total structure function F_2 for Q^2 between 1.5 GeV^2 and 5000 GeV^2 and x between 3×10^{-5} and 0.32 are now available [6]. As for the charm structure function, there are preliminary low–x data on F_2^c at $Q^2=13$ GeV^2, 23 GeV^2 and 50 GeV^2 with rather large errors [7].

Using the first of the inequalities (56) we get (assuming $F_2^c(m_c^2,x)$, $F_2^b(m_c^2,x)\simeq 0$ (cf. [8]))

$$F_2(Q^2,x)>0.4\left[F_2(Q^2,x)-F_2^b(Q^2,x)-F_2(m_c^2,x)\right]. \tag{57}$$

Let us estimate $F_2(Q^2, x)$ from below for $x = 5 \times 10^{-3}$ and $x = 5 \times 10^{-4}$ and several values of Q^2. There are data on $F_2(Q^2, x)$ for $Q^2 = 2.5$ GeV^2, $x = 4 \times 10^{-3}$ and $Q^2 = 2.5$ GeV^2, $x = 6.3 \times 10^{-4}$ [6]. The relative bottom contribution, F_2^b/F_2, reaches at most 2 to 3% at HERA.

Putting $F_2(m_c^2, 4.0 \times 10^{-3}) \simeq F_2(m_c^2, 5.0 \times 10^{-3})$ and $F_2(m_c^2, 6.3 \times 10^{-4}) \simeq F_2(m_c^2, 5.0 \times 10^{-4})$ and choosing $m_c = 1.58$ GeV we obtain from (57) the low bounds for F_2^c presented in Tables 1 and 2.

Q^2, GeV^2	12	25	45
$F_2^c(Q^2, x)$	0.106 ± 0.054	0.139 ± 0.044	0.195 ± 0.046

Table 1: The low bounds on $F_2^c(Q^2, x)$ for $x = 5 \times 10^{-3}$.

Q^2, GeV^2	12	25
$F_2^c(Q^2, x)$	0.207 ± 0.063	0.355 ± 0.070

Table 2: The low bounds on $F_2^c(Q^2, x)$ for $x = 5 \times 10^{-4}$.

These quantitative estimates of F_2^c do not contradict the preliminary data on the charm contribution to F_2 [7]. For a detailed comparison of our predictions with the data, an improved measurement of the charm component F_2^c is required.

Conclusions

In this paper we have demonstrated that the lowest–order quark loop contributions to the structure functions at small x contain mass–dependent terms which scale at high Q^2. This effect can be observed experimentally, and we predict theoretical bounds for the corresponding contributions from c and b–quarks (see Eqs. (55) and (56), Tables 1 and 2).

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Appendix I

In this section we calculate an imaginary part of the photon–gluon amplitude averaged in the photon Lorentz indices with the tensors $g_{\mu\nu}$ and $p_\mu p_\nu$ (see Eqs. (12)).

Let us consider the function $C^{(a)}_{\alpha\beta}$. In the first order in the strong coupling α_s (a one–loop approximation), it can be represented in the form

$$\begin{aligned} C^{(a)}_{\alpha\beta}(Q^2,k^2,qk) &= \frac{\alpha_s}{\pi^2}\int d^4r[\frac{1}{l^4}I^1_{\alpha\beta}(r,q,k;m^2) + \frac{1}{l^2(2qk-l^2)}I^2_{\alpha\beta}(r,q,k,;m^2)] \\ &\times\ \delta_+((q-r)^2-m^2)\delta_+((k+r)^2-m^2), \end{aligned} \tag{I.1}$$

where

$$\begin{aligned} I^1_{\alpha\beta} = &- g_{\alpha\beta}\frac{1}{2}[(l^4-2l^2qk-k^2Q^2)+2m^2k^2] + r_\alpha r_\beta 2(Q^2-2m^2) \\ &+ (r_\alpha k_\beta + k_\alpha r_\beta)(Q^2-2m^2) - (r_\alpha q_\beta + r_\alpha q_\beta)l^2 \\ &- (k_\alpha q_\beta + k_\alpha q_\beta)l^2 \end{aligned} \tag{I.2}$$

corresponds to a contribution of a ladder diagram, while

$$\begin{aligned} I^2_{\alpha\beta} &= g_{\alpha\beta}\frac{1}{2}k^2[(k^2-Q^2+2qk)-2m^2] + r_\alpha r_\beta 2[(Q^2+k^2)-2m^2] \\ &- k_\alpha k_\beta(l^2-Q^2) + q_\alpha q_\beta(l^2+k^2) \\ &- (r_\alpha k_\beta + k_\alpha r_\beta)[(l^2-Q^2-k^2-qk)+2m^2] \\ &- (r_\alpha q_\beta + r_\alpha q_\beta)[(l^2+Q^2-k^2-qk)-2m^2] \\ &- (k_\alpha q_\beta + k_\alpha q_\beta)\frac{1}{2}[(Q^2+k^2)-2m^2] \end{aligned} \tag{I.3}$$

is a contribution of a crossed one. In Eqs. (I.2) and (I.3) we denote

$$l^2 = m^2 - r^2. \tag{I.4}$$

The following equality thus takes place

$$\begin{aligned} &\int d^4r r_\alpha f(r^2)\delta_+((q-r)^2-m^2)\delta_+((k+r)^2-m^2) \\ &= \frac{\pi}{4\sqrt{D}}[k_\alpha A_1(Q^2,k^2,qk;m^2) + q_\alpha A_2(Q^2,k^2,qk;m^2)], \end{aligned} \tag{I.5}$$

where

$$\begin{aligned} A_1 &= -\frac{1}{2D}\int\limits_{l^2_-}^{l^2_+} dl^2[Q^2(qk+k^2)+l^2(qk-Q^2)]f(l^2), \\ A_2 &= \frac{1}{2D}\int\limits_{l^2_-}^{l^2_+} dl^2[l^2(qk+k^2)-k^2(qk-Q^2)]f(l^2) \end{aligned} \tag{I.6}$$

with

$$
\begin{aligned}
l_\pm^2 &= (qk) \pm \left[D(1-\frac{4m^2}{s})\right]^{1/2}, \\
D &= (qk)^2 + Q^2k^2, \\
s &= k^2 - Q^2 + 2qk.
\end{aligned} \tag{I.7}
$$

Analogously, we have

$$
\begin{aligned}
&\int d^4r r_\alpha r_\beta f(r^2)\delta_+((q-r)^2-m^2)\delta_+((k+r)^2-m^2) \\
&= \frac{\pi}{4\sqrt{D}}[g_{\alpha\beta}B_1 + k_\alpha k_\beta B_2 + q_\alpha q_\beta B_3 + \frac{1}{2}(k_\alpha q_\beta + q_\alpha k_\beta)B_4],
\end{aligned} \tag{I.8}
$$

where

$$
\begin{aligned}
B_1 &= \int\limits_{l_-^2}^{l_+^2} dl^2 \Big\{\frac{1}{8D}Rs + \frac{1}{2}m^2\Big\} f(l^2), \\
B_2 &= \int\limits_{l_-^2}^{l_+^2} dl^2 \Big\{-\frac{3}{8D^2}Q^2Rs + \frac{1}{4D}[(l^2-Q^2)^2 - 2m^2Q^2]\Big\} f(l^2), \\
B_3 &= \int\limits_{l_-^2}^{l_+^2} dl^2 \Big\{\frac{3}{8D^2}k^2Rs + \frac{1}{4D}[(l^2+k^2)^2 + 2m^2k^2]\Big\} f(l^2), \\
B_4 &= \int\limits_{l_-^2}^{l_+^2} dl^2 \Big\{-\frac{3}{4D^2}qkRs + \frac{1}{2D}[R - l^2s - 2m^2qk]\Big\} f(l^2).
\end{aligned} \tag{I.9}
$$

In Eqs. (I.9) a notation

$$
R = l^4 - 2l^2qk - k^2Q^2 \tag{I.10}
$$

is introduced.

Accounting for all that was said above, we get

$$
C_{\alpha\beta}^{(a)}(Q^2,k^2,qk) = \frac{\alpha_s}{4\pi}\frac{1}{\sqrt{D}}\int\limits_{l_-^2}^{l_+^2} dl^2 \Big[\frac{1}{l^4}\tilde{I}_{\alpha\beta}^1(l^2,q,k;m^2) + \frac{1}{l^2(2qk-l^2)}\tilde{I}_{\alpha\beta}^2(l^2,q,k,;m^2)\Big] \tag{I.11}
$$

with

$$
\begin{aligned}
\tilde{I}_{\alpha\beta}^1 = &- \frac{1}{2}\Big\{g_{\alpha\beta}\Big[1-\frac{1}{2D}Q^2s\Big]R - k_\alpha k_\beta\frac{Q^2}{D}\Big[R - Q^2s - \frac{3}{2D}Q^2sR\Big] \\
&+ q_\alpha q_\beta\frac{1}{D}\Big[Rk^2 + l^4s - \frac{3}{2D}k^2Q^2sR\Big]
\end{aligned}
$$

$$
\begin{aligned}
&- (k_\alpha q_\beta + q_\alpha k_\beta)\frac{1}{D}\Big[Rqk - l^2Q^2s - \frac{3}{2D}Q^2qksR\Big]\Big\} \\
&- m^2\Big\{g_{\alpha\beta}\Big[(k^2-Q^2) + \frac{1}{2D}sR\Big] \\
&+ k_\alpha k_\beta\frac{1}{D}\Big[R - Q^2s + Q^4 - \frac{3}{2D}Q^2sR\Big] \\
&+ q_\alpha q_\beta\frac{1}{D}\Big[(l^2+k^2)^2 - k^2Q^2 + \frac{3}{2D}k^2sR\Big] \\
&+ (k_\alpha q_\beta + q_\alpha k_\beta)\frac{1}{D}\Big[R - (l^2+k^2)(qk-Q^2) + Q^2qk - \frac{3}{2D}qksR\Big]\Big\} \\
&- 2m^4\Big\{g_{\alpha\beta} - k_\alpha k_\beta\frac{Q^2}{D} + q_\alpha q_\beta\frac{k^2}{D} - (k_\alpha q_\beta + q_\alpha k_\beta)\frac{qk}{D}\Big\} \qquad \text{(I.12)}
\end{aligned}
$$

and

$$
\begin{aligned}
\tilde{I}^2_{\alpha\beta} \;=\; &\frac{s}{2}\Big\{g_{\alpha\beta}\Big[k^2 + \frac{1}{2D}(Q^2+k^2)R\Big] \\
&+ k_\alpha k_\beta\frac{1}{D}\Big[R - Q^2(Q^2+k^2) - \frac{3}{2D}Q^2(Q^2+k^2)R\Big] \\
&- q_\alpha q_\beta\frac{1}{D}\Big[R - k^2(Q^2+k^2) - \frac{3}{2D}k^2(Q^2+k^2)R\Big] \\
&- (k_\alpha q_\beta + q_\alpha k_\beta)\frac{1}{D}qk(Q^2+k^2)\Big[1 + \frac{3}{2D}R\Big]\Big\} \\
&- m^2\Big\{-g_{\alpha\beta}\Big[Q^2 - \frac{1}{2D}sR\Big] + k_\alpha k_\beta\frac{1}{D}\Big[R - 2Q^2(qk-Q^2) - \frac{3}{2D}Q^2sR\Big] \\
&+ q_\alpha q_\beta\frac{1}{D}\Big[R + 2k^2(qk-Q^2) + \frac{3}{2D}k^2sR\Big] \\
&+ (k_\alpha q_\beta + q_\alpha k_\beta)\frac{1}{D}\Big[R - (qk)^2 + 2qkQ^2 + Q^2k^2 - \frac{3}{2D}qksR\Big]\Big\} \\
&- 2m^4\Big\{g_{\alpha\beta} - k_\alpha k_\beta\frac{Q^2}{D} + q_\alpha q_\beta\frac{k^2}{D} - (k_\alpha q_\beta + q_\alpha k_\beta)\frac{qk}{D}\Big\}. \qquad \text{(I.13)}
\end{aligned}
$$

Equation (I.11) can be represented in the form

$$
\begin{aligned}
C^{(a)}_{\alpha\beta} \;=\; &A_{\alpha\beta}\frac{\alpha_s}{4\pi}\frac{1}{\sqrt{D}}\int\limits_{l^2_-}^{l^2_+} dl^2\Big\{\frac{1}{l^2}\Big[qk + \frac{1}{4Dqk}(k^2-Q^2)\Big(2(qk)^2 + k^2Q^2\Big)s\Big] \\
&+ \frac{1}{l^2}\Big[Q^2 + \frac{1}{2D}\Big(2(qk)^2 + k^2Q^2\Big)s\Big]\frac{m^2}{qk} - \frac{2}{l^2}\frac{m^4}{qk} \\
&+ \frac{1}{2l^4}\Big[1 - \frac{1}{2D}Q^2s\Big]k^2Q^2 - \frac{1}{l^4}\Big[k^2 - Q^2 - \frac{1}{2D}k^2Q^2s\Big]m^2 \\
&- \frac{2}{l^4}m^4 - \frac{1}{2}\Big[1 + \frac{1}{2D}k^2s\Big]\Big\}
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad B_{\alpha\beta}\frac{\alpha_s}{4\pi}\frac{1}{\sqrt{D}}\int\limits_{l_-^2}^{l_+^2} dl^2\Big\{\frac{1}{l^2}\Big[qk+\frac{1}{4Dqk}(k^2-Q^2)\Big(2(qk)^2-k^2Q^2\Big)s\Big] \\
&+ \quad \frac{1}{l^2}\Big[-4qk+3Q^2+\frac{3}{2D}\Big(2(qk)^2+k^2Q^2\Big)s\Big]\frac{m^2}{qk}-\frac{2}{l^2}\frac{m^4}{qk} \\
&+ \quad \frac{1}{2l^4}\Big[1-\frac{3}{2D}Q^2s\Big]k^2Q^2-\frac{1}{l^4}\Big[k^2-Q^2-\frac{3}{2D}k^2Q^2s\Big]m^2 \\
&- \quad \frac{2}{l^4}m^4-\frac{1}{2}\Big[1+\frac{3}{2D}k^2s\Big]\Big\}, \qquad\qquad \text{(I.14)}
\end{aligned}
$$

where

$$
\begin{aligned}
A_{\alpha\beta} &= \Big(g_{\alpha\beta}-\frac{k_\alpha k_\beta}{k^2}\Big), \\
B_{\alpha\beta} &= (q_\alpha-k_\alpha\frac{qk}{k^2})(q_\beta-k_\beta\frac{qk}{k^2})\frac{k^2}{D}. \qquad\qquad \text{(I.15)}
\end{aligned}
$$

From Eqs. (I.14) and (I.15) one can see that $C^{(a)}_{\alpha\beta}$ obeys a condition

$$
k^\alpha C^{(a)}_{\alpha\beta}=0. \qquad\qquad \text{(I.16)}
$$

In deriving (I.14) from (I.11)–(I.13) we took into account that $l^2_\pm \to l^2_\mp$ at $l^2 \to 2qk-l^2$, $l^2_\pm$ being defined by Eq. (I.7).

Note that the difference of the structure function under consideration, ΔF_2, is defined by $\Delta C^{(r)}_{\alpha\beta}$ $(r=a,b)$. The analysis shows that a leading contribution into $\Delta C^{(r)}_{\alpha\beta}$ comes from the region $-k^2 \sim m^2 \ll Q^2$. Rejecting terms $\mathrm{O}(k^2/Q^2)$ and $\mathrm{O}(m^2/Q^2)$, we obtain the formula

$$
\begin{aligned}
C^{(a)}_{\alpha\beta} &= A_{\alpha\beta}\frac{\alpha_s}{4\pi}\int\limits_{l_-^2}^{l_+^2} dl^2\Big[\frac{1}{l^2}\Big(1-\frac{1}{2(qk)^2}Q^2s\Big)+\frac{1}{2l^4}\frac{k^2Q^2}{qk}\Big(1-\frac{1}{2(qk)^2}Q^2s+2\frac{m^2}{k^2}\Big) \\
&- \frac{1}{2(qk)}\Big]+B_{\alpha\beta}\frac{\alpha_s}{4\pi}\int\limits_{l_-^2}^{l_+^2} dl^2\Big[\frac{1}{l^2}\Big(1-\frac{1}{2(qk)^2}Q^2s\Big) \\
&+ \frac{1}{2l^4}\frac{k^2Q^2}{qk}\Big(1-\frac{3}{2(qk)^2}Q^2s+2\frac{m^2}{k^2}\Big)-\frac{1}{2(qk)}\Big]. \qquad\qquad \text{(I.17)}
\end{aligned}
$$

Now let us consider the function $C^{(b)}_{\alpha\beta}$ (see Eqs. (12)). The result of our calculations is of the form

$$
\begin{aligned}
C^{(b)}_{\alpha\beta}(Q^2,k^2,qk) &= \frac{3\alpha_s}{2\pi^2}\frac{Q^2}{D}\int d^4r\Big[\frac{1}{l^4}J^1_{\alpha\beta}(r,q,k;m^2)+\frac{1}{l^2(2qk-l^2)}J^2_{\alpha\beta}(r,q,k,;m^2)\Big] \\
&\times \; \delta_+((q-r)^2-m^2)\delta_+((k+r)^2-m^2), \qquad\qquad \text{(I.18)}
\end{aligned}
$$

where the contributions from ladder and crossed diagrams are, respectively,

$$
\begin{aligned}
J^1_{\alpha\beta} &= g_{\alpha\beta}\frac{1}{2}k^4 s + r_\alpha r_\beta 2[R - k^2(2l^2 - Q^2) + k^2 s)] \\
&+ k_\alpha k_\beta 2[R - k^2(l^2 - Q^2)] \\
&+ (r_\alpha k_\beta + k_\alpha r_\beta)[2R - k^2(3l^2 - 2Q^2) + k^2 s] \\
&+ (r_\alpha q_\beta + r_\alpha q_\beta)l^2 k^2 + (k_\alpha q_\beta + k_\alpha q_\beta)l^2 k^2
\end{aligned}
\tag{I.19}
$$

and

$$
\begin{aligned}
J^2_{\alpha\beta} = &- g_{\alpha\beta}\frac{1}{2}k^4 s + r_\alpha r_\beta 2(R - k^4) + k_\alpha k_\beta (l^2 - Q^2)k^2 \\
&- q_\alpha q_\beta (l^2 + k^2)k^2 + (r_\alpha k_\beta + k_\alpha r_\beta)[R + k^2(l^2 - k^2 - qk)] \\
&- (r_\alpha q_\beta + r_\alpha q_\beta)[R - k^2(l^2 + k^2 - qk)] \\
&- (k_\alpha q_\beta + k_\alpha q_\beta)\frac{1}{2}[2R - k^2(k^2 - Q^2)].
\end{aligned}
\tag{I.20}
$$

Using formulae (I.5) and (I.8) we obtain

$$
\begin{aligned}
C^{(b)}_{\alpha\beta} = &- A_{\alpha\beta}\frac{3\alpha_s}{8\pi}\frac{Q^2}{D^{3/2}}\int\limits_{l_-^2}^{l_+^2} dl^2 \Bigg\{ \frac{1}{4l^2}\Bigg[\frac{1}{qk}(k^2 - Q^2) \\
&+ \frac{1}{D}\Big(qk(3k^2 - 3Q^2 + 4qk) - 2k^2Q^2\Big)\Bigg]k^2 s \\
&+ \frac{1}{l^2}\Big[2(k^2 + qk) + \frac{1}{qk}k^2(k^2 + Q^2)\Big]m^2 \\
&- \frac{1}{2l^4}\Big[1 - \frac{1}{2D}Q^2 s\Big]k^4 s - \frac{1}{l^4}k^2 s m^2 \\
&+ l^2\frac{1}{2D}(k^2 + qk)s - \frac{1}{2D}(k^2 + qk)(k^2 + 2qk)s\Bigg\} \\
&- B_{\alpha\beta}\frac{3\alpha_s}{8\pi}\frac{Q^2}{D^{3/2}}\int\limits_{l_-^2}^{l_+^2} dl^2 \Bigg\{ \frac{1}{4l^2}\Bigg[-\frac{1}{qk}(k^2 - Q^2) \\
&+ \frac{3}{D}\Big(qk(3k^2 - 3Q^2 + 4qk) - 2k^2Q^2\Big)\Bigg]k^2 s \\
&+ \frac{1}{l^2}\Big[2(k^2 + qk) + \frac{1}{qk}k^2(k^2 + Q^2)\Big]m^2 \\
&- \frac{1}{2l^4}\Big[1 - \frac{3}{2D}Q^2 s\Big]k^4 s - \frac{1}{l^4}k^2 s m^2 + l^2\frac{3}{2D}(k^2 + qk)s \\
&- \frac{3}{2D}(k^2 + qk)(k^2 + 2qk)s + s\Bigg\}.
\end{aligned}
\tag{I.21}
$$

At $-k^2 \sim m^2 \ll Q^2$ we get from (I.21)

$$\begin{aligned}
C^{(b)}_{\alpha\beta} = & - A_{\alpha\beta}\frac{3\alpha_s}{8\pi}\frac{Q^2}{(qk)^2}\int\limits_{l^2_-}^{l^2_+} dl^2 \left\{\frac{1}{l^2}\left[\frac{(qk-Q^2)s}{(qk)^2}+\frac{2m^2}{k^2}\right]k^2\right. \\
& - \left.\frac{1}{2l^4}\frac{k^4 s}{qk}\left[1-\frac{Q^2 s}{2(qk)^2}+\frac{2m^2}{k^2}\right]-\frac{s}{qk}+l^2\frac{s}{2(qk)^2}\right\} \\
& - B_{\alpha\beta}\frac{3\alpha_s}{8\pi}\frac{Q^2}{(qk)^2}\int\limits_{l^2_-}^{l^2_+} dl^2 \left\{\frac{1}{l^2}\left[\frac{(3qk-2Q^2)s}{(qk)^2}+\frac{2m^2}{k^2}\right]k^2\right. \\
& - \left.\frac{1}{2l^4}\frac{k^4 s}{qk}\left[1-\frac{3Q^2 s}{2(qk)^2}+\frac{2m^2}{k^2}\right]-\frac{2s}{qk}+l^2\frac{3s}{2(qk)^2}\right\}.
\end{aligned} \tag{I.22}$$

When calculating $C^{(b)}_{\alpha\beta}$ we used the replacement

$$\frac{p_\alpha p_\beta}{\sqrt{(pq)^2+Q^2M^2}} \to \frac{k_\alpha k_\beta}{\sqrt{(qk)^2+Q^2k^2}}. \tag{I.23}$$

This results in power corrections that are, however, ignored everywhere in our consideration.

Appendix II

In anology with quark distribution, scalar gluon distribution inside the nucleon, D_g, can be defined by considering deep inelastic scattering with scalar gauge–invariant gluonic currents

$$J(x) = \frac{1}{4}(G^a_{\mu\nu}(x))^2. \tag{II.1}$$

Let us define D_g via a gluonic structure function F:

$$D(Q^2, x) = \frac{1}{x}F(Q^2, x) = \frac{1}{4\pi}\mathrm{Disc}T, \tag{II.2}$$

where

$$T = i\int d^4z \exp(iqz)\langle p|TJ(z)J(0)|p\rangle. \tag{II.3}$$

Function F is given by the formula

$$\frac{1}{x}F = \frac{1}{4\pi}\int \frac{d^4k}{(2\pi)^4}\Big[\Pi_{\alpha\beta}(q,k)\frac{1}{k^4}\tilde{\Gamma}^{\alpha\beta}(k,p)\Big]. \tag{II.4}$$

Here $\Pi_{\alpha\beta}$ is a gluonic partonometer

$$\Pi_{\alpha\beta} = -g_{\alpha\beta}2\pi\delta((q+k)^2)Q^2, \tag{II.5}$$

while $\tilde{\Gamma}^{\alpha\beta}$ means an imaginary part of the virtual gluon–nucleon amplitude with tensor parts of the gluonic propagators $d_{\alpha\alpha'}$ included. So, we have by definition

$$\tilde{\Gamma}_{\alpha\beta} = d_{\alpha\alpha'}(k)d_{\beta\beta'}(k)\Gamma^{\alpha'\beta'}, \tag{II.6}$$

where the gluon distribution $\Gamma_{\alpha\beta}$ enters Eq. (4).

Tensor $\tilde{\Gamma}_{\alpha\beta}$ has the following Lorentz structure

$$\tilde{\Gamma}_{\alpha\beta} = d_{\alpha\beta}(p)\tilde{\Gamma}_1 + d_{\alpha\beta}(k)\tilde{\Gamma}_2 + \Big(p_\alpha - k_\alpha\frac{pn}{kn}\Big)\Big(p_\beta - k_\beta\frac{pn}{kn}\Big)\frac{1}{k^2}\tilde{\Gamma}_3 + n_\alpha n_\beta\frac{k^2}{(kn)^2}\tilde{\Gamma}_4, \tag{II.7}$$

with $\tilde{\Gamma}_i = \tilde{\Gamma}_i(k^2, M^2, pk)$. Then from Eqs. (II.6), (II.7) and (13) the relation between $\tilde{\Gamma}_i$ and Γ_i can easily be obtained. In particular, we have

$$\tilde{\Gamma}_3 = \Gamma_2 + \Gamma_4. \tag{II.8}$$

If we substitute (II.5), (II.7) into (II.4), we get in LLA

$$D(Q^2, Q_0^2, x) = \int_{Q_0^2}^{Q^2}\frac{dk^2}{k^4}\int\frac{d^2k_\perp}{2(2\pi)^4}\Big[\tilde{\Gamma}_1(k^2,k_\perp,x) + \tilde{\Gamma}_2(k^2,k_\perp,x) + \frac{1-x}{2x^2}\tilde{\Gamma}_3(k^2,k_\perp,x)\Big]. \tag{II.9}$$

As can be seen, at small x the gluon distribution (II.9) is mainly defined by the invariant function $\tilde{\Gamma}_3$ (or, equivalently, by the combination $\Gamma_2 + \Gamma_4$).

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Figure Captions

Figs. 1a-1d: $\Delta C(\eta, y)$ as a function of the variable η at several fixed values of y.

Figs. 2a-2d: $C(\eta, y)$ (continuous curves) and $\Delta C(\eta, y)$ (dashed curves) as functions of the variable η ($\eta \geq 0$) at several fixed values of y.